On Fast Transmission Topology Control Heuristics

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Abstract—The standard optimal power flow (OPF) problem minimizes generation costs over one study period assuming a fixed system topology. The prospect of a smart grid incorporating extensive cyber capabilities enabling significant progress in economic efficiency, reliability and environmental sustainability, ought to transform the OPF problem accordingly. This paper discusses the inclusion of tractable dynamic transmission topology control in the OPF problem based on heuristic control policies derived from individual transmission "line profit" criteria. Simulations on the IEEE 118-bus test system demonstrate the effectiveness of the heuristic policies in reducing production costs. As the algorithm's requisite information to identify promising candidate elements for switching is standard output of the OPF solution, the computational effort is up to four orders of magnitude better than dynamic transmission topology control performance reported in the literature.

Index Terms— Transmission congestion, topology control, transmission switching, optimal power flow, renewable integration.

I. INTRODUCTION

RADITIONALLY, power system operational decision making has been based on a static, exogenously fixed, pre-contingency transmission topology. In fact, the standard optimal power flow (OPF) problem is limited to the minimization of generation costs over a single period subject to exogenously-determined system loads, reserve requirements, and system topology. National directives, such as the National Energy Policy Act and the Energy Independence and Security Act, seek to incorporate new technologies into the energy grid with the aim of increased economic efficiency, reliability and environmental sustainability. This suggests that the formulation of the OPF problem should be transformed to take advantage of the intelligence embedded in the smart grid platform, and regard as flexible decision variables what have thus far been considered problem parameters, e.g., loads, reserve requirements, and transmission topology. The desirable transformation should increase demand-side participation in power markets on a par basis with dispatchable generation to increase consumer welfare, provide valuable reserve capacity, and reduce generation costs [1], [2], [3]. Large-scale integration of variable renewable generation will increase ancillary service requirements [4], rendering reserve and regulation requirements a function of volatile generation bids and demand uncertainty [5]. Finally, dynamic control of the underlying network topology, which is desirable under present conditions, will be even more important under a future with flexible

demand and significant volatile renewable generation. Incorporating transmission topology control in the OPF problem while maintaining problem tractability is the topic of this paper.

Although optimal long-term transmission planning addresses reliability and production cost minimization, its timescale (years) is vastly different from that of transmission topology control (hours), and, more importantly, the information about the state of and requirements on the transmission system are vastly different as well. Transmission planning aims at allowing sufficient long-term capacity, including system redundancies designed for reliability that are robust over geographically-specific load and generation growth forecasts. Transmission topology control responds to the revelation of uncertainty handled in short-term markets that schedule generation and reserves on an hourly basis subject to transmission constraints. The separate, and sometimes conflicting, goals of transmission planning and economic dispatch result in situations where lines built for reliability result in economic inefficiency [6]. Khodaei et al. [7] present a transmission switching coordinated expansion planning model in order to link the two problems. Historically, due to the ability of forecasters to accurately project seasonal-levels of generation and load, there has been minimal energy transfer variability. However, with the restructuring of markets and the impending large-scale integration of renewable generation, this variation is likely to dramatically increase.

The issue of branches impairing system performance was first discussed in 1968 in a transportation network setting, and is known as Braess's Paradox [8]. Braess's Paradox states that an additional link in a transportation network can sometimes increase congestion. This game-theoretic result occurs when each user of the transportation network chooses their route selfishly. This paradoxs undesirable congestion can be averted by introducing a cost of using a joint resource or by routing transportation network users centrally, the latter, resulting in a traditional network max-flow problem. However, in AC electricity transmission, flows are typically not routable¹. Instead, Kirchoff's Laws determine power flows over transmission lines making congestion avoidance require out of merit generation dispatch.

Corrective switching, including transmission line, bus-bar, and shunt element switching, has been a topic of research since the 1980s, typically considered as a means to deal with line overloads and voltage violations either pre- [9] or postcontingency [10], [11]. Other work incorporated transmission topology control after solving the OPF problem, using the resultant dispatch as a problem input to either reduce losses [12],

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¹Except for the few branches that have flow control devices, such as phase shifters.

[13] or line and transformer congestion [14]. Fisher et al. [15], report that transmission topology control is already employed by system operators to deal with contingency, voltage profile, reactive power consumption, reliability and maintenance issues. In fact, it appears that transmission switches are operated for many reasons – except for economic ones.

Recently, dynamic optimization of transmission topology within the OPF problem has become popular [15], [16], [17], [18], [19]. Fisher et al. [15], [17] report 25% generation cost savings through transmission topology control of the 118bus IEEE test system. They solved the mixed integer linear program (MIP) dc optimal power flow (DCOPF) to near optimality by using an iterative partitioning and parallel solution approach. Their approach proved computationally expensive with solution times in excess of 30 minutes without attaining optimality, and is hence almost certain to be intractable for real-size systems which often contain more than 10,000 buses. However, Fisher et al. [15] found that the majority of cost savings occurred as a result of removing relatively few lines. What is believed to be a near optimal solution involved 38 open transmission lines. Yet 22% cost savings were achieved by opening only 7 lines.

This paper presents an algorithm that improves the transmission topology configuration relying on a set of heuristic criteria grounded on the global marginal cost of congestion, which is routinely estimated by the traditional OPF solution. We use engineering judgment to translate the global marginal cost of congestion to the small set of lines associated with the lion's share of cost savings. In addition, rather than solving iterative MIPs, we determine topology improvements by iteratively solving the linear programming (LP) formulation of the DCOPF, thus achieving economic efficiency with minimal computational effort. Implementation on the IEEE 118-bus test system shows that our heuristic algorithm decreases the initial congestion cost by 2/3 with CPU times well below one second.

The paper is organized as follows. Section II provides intuition on the economic transmission topology control problem. Section III formally presents the flow and dispatch models and characterizes the congestion rent. Section IV discusses a general structure for transmission topology control heuristics, Section V presents our proposed heuristic, and Section VI presents computational results on the IEEE 118-bus test network. Section VII concludes and briefly describes the direction of future work. The Appendix includes the implemented IEEE 118-bus test system generation model.

II. ECONOMIC TRANSMISSION TOPOLOGY CONTROL

The total transfer capability (TTC) between nodes (or areas, zones) of a transmission system measures the ability of the network to move power (MW) between nodes or groups of nodes (areas) [20], [21]. TTC can be calculated using an OPF approach [21], where the maximum amount of flow from a source node or area to a sink node or area is computed, analogous to the maximum network flow problem. However, as previously mentioned, since power flow is dictated by Kirchoff's Laws, the TTC is less than the total transmission

capacity, defined as the aggregate thermal capacity of all lines between the two nodes in the network. In fact, TTCs are dependent on generation, customer demand, and transmission system conditions for each time period.

Due to transmission limits, a strict merit-order dispatch of generation resources is typically not feasible. Nodal prices, computed using shadow prices revealed in the OPF, indicate the marginal cost of providing power to that node, given the costs of available generators and transmission network constraints. Since losses are minimal in the high voltage transmission network, the majority of the variation in nodal prices is due to congestion pricing. Congestion prices result when transmission constraints prevent lower cost generators from being fully utilized. As a result, higher cost generation needs to be dispatched to serve load.

Kirchoff's Laws state that power flow along all paths between two nodes are inversely proportional to the impedance of each path. Therefore, congestion on one path can determine the TTC between two nodes, even if excess thermal capacity exists in all other paths. Lines that limit the TTC, and hence the ability for economic dispatch, are potentially desirable to disconnect, while disconnected lines whose addition would yield another path are potentially desirable to connect. However, it is important to note that this problem is highly dynamic in nature. Consider for instance a power control area with high penetration of both wind and solar plants. These generation sources will not be co-located and will be typically generating during different times of the day, and be varying from day to day. In addition, the location of loads changes as people go to work in the morning and return home in the early evening. As such, it is evident that optimal topology control is a dynamic maximization as the nodes between which it is optimal to increase the TTC change many times throughout the day.

III. POWER FLOWS, OPTIMAL DISPATCH, AND CONGESTION MODELS

Consider a power system in which the linearized lossless dc assumptions hold. This system has buses n = 1, ..., Nand lines $\ell = 1, ..., L$. Each line ℓ is associated with an ordered pair of nodes (m_{ℓ}, n_{ℓ}) , with the convention that the flow direction of line ℓ is from node m_{ℓ} and to node n_{ℓ} . Let bus N be the reference bus, which has voltage angle 0. Let $\tilde{\mathbf{B}}$ be the branch susceptance matrix, a diagonal matrix with the line susceptances as its elements. Let \mathbf{A} be the reduced incidence matrix, an $L \times (N-1)$ matrix which for each row ℓ has elements -1 and 1 in the columns corresponding to the from and to nodes of line ℓ , respectively, and 0 for all other nodes, except for each line that is connected to the reference bus, which only has an entry corresponding to the bus that is not the reference bus. The reduced nodal susceptance matrix \mathbf{B} is given by

$$\mathbf{B} = -\mathbf{A}'\mathbf{B}\mathbf{A}.\tag{1}$$

The nodal power balance equations, which state that the net load at each bus equals the net line flow to the bus, can be expressed in terms of the vector of power flows \mathbf{f} on each transmission line and \mathbf{A} as

$$\mathbf{l} - \mathbf{p}) = \mathbf{A}' \mathbf{f},\tag{2}$$

where \mathbf{p} and \mathbf{l} are the vectors of nodal power generation and loads, respectively. The power flow \mathbf{f} on each transmission line is

$$\mathbf{f} = \mathbf{B}\mathbf{A}\boldsymbol{\theta},\tag{3}$$

where θ is the vector of nodal voltage angles.

From (2) and (3), the well-known nodal power equations are obtained,

$$(\mathbf{p} - \mathbf{l}) = \mathbf{B}\boldsymbol{\theta}.\tag{4}$$

From (3) and (4), the power flows can be expressed as an explicit function of the loads and generation,

$$\mathbf{f} = \tilde{\mathbf{B}}\mathbf{A}\mathbf{B}^{-1}(\mathbf{p} - \mathbf{l}) \tag{5}$$

$$= \Psi(\mathbf{p} - \mathbf{l}). \tag{6}$$

The transmission sensitivity matrix Ψ [22], also known as the *injection shift factor matrix*, gives the variations in flows due to changes in the nodal injections, with the reference bus assumed to ensure the real power balance. The shift factor matrix is a function of the characteristics of the transmission elements and of the state of the transmission switches. This dependence, however, is usually not made explicit in OPF formulations, since the assumption is that the transmission topology is fixed.

For a given point in time, the system operator dispatches the committed units so as to minimize the total costs of operations. Assume that the generation costs are piecewise linear, and denote the vector of nodal generation variable costs in \$/MWh by c. The economic dispatch solved is a linearized lossless DC OPF [23],

$$\mathcal{C} = \min_{\mathbf{p}} \mathbf{c}' \mathbf{p} \tag{7}$$

s.t.
$$\mathbf{1'p} = \mathbf{1'l} \leftrightarrow \lambda$$
 (8)

$$\underline{\mathbf{f}} \leq \Psi(\mathbf{p} - \mathbf{l}) \leq \overline{\mathbf{f}} \quad \leftrightarrow \quad \underline{\boldsymbol{\mu}}, \overline{\boldsymbol{\mu}} \tag{9}$$

$$\underline{\mathbf{p}} \le \mathbf{p} \le \overline{\mathbf{p}} \quad \leftrightarrow \quad \underline{\gamma}, \overline{\gamma}. \tag{10}$$

Constraint (8) ensures the total load-generation balance, (9) enforces the flow limits on transmission elements and flowgates, where lower limits usually represent the limit in the opposite flow direction, and (10) models the lower and upper generation limits. The nodal prices are given by

$$\boldsymbol{\pi} = -\big(\lambda \mathbf{1} + \boldsymbol{\Psi}'(\boldsymbol{\overline{\mu}} - \boldsymbol{\mu})\big). \tag{11}$$

The marginal units are those that may change their output at an optimal re-dispatch in response to a small variation of a system parameter. They consist of the units that are dispatched between their generating limits, and of those that are at the limit but with the corresponding dual variables $\overline{\gamma}$ and $\underline{\gamma}$ equal to 0. These units have their nodal prices equal to their costs, and usually the number of marginal generators is equal to the number of binding transmission constraints plus one.

As a benchmark, it is useful to consider the transmissionunconstrained OPF, which has the same objective function as (7)-(10), but does not enforce (9). Denote the optimal cost of this OPF by \underline{C} , where the underline indicates that $\underline{C} \leq C$. The *congestion cost* is defined as the production cost increase due to the transmission constraint enforcement, $C - \underline{C}$.

In the lossless DCOPF, the *congestion rent* \mathcal{K} , also called *merchandising surplus*, is defined as the rent the system

operator obtains if each load pays its nodal price for its consumption and each generator is paid the nodal price for its production [24],

$$\mathcal{K} := \pi' (\mathbf{l} - \mathbf{p}). \tag{12}$$

The congestion rent is equal to the sum of the product of the line flows times the shadow price of the line constraints,

$$\mathcal{K} = -\pi'(\mathbf{p} - \mathbf{l}) \tag{13}$$

$$= (\lambda \mathbf{1} + \mathbf{\Psi}'(\overline{\boldsymbol{\mu}} - \boldsymbol{\mu}))'(\mathbf{p} - \mathbf{l})$$
(14)

$$= (\overline{\mu} - \mu)' \Psi(\mathbf{p} - \mathbf{l}) \tag{15}$$

$$= (\overline{\boldsymbol{\mu}} - \boldsymbol{\mu})' \mathbf{f}. \tag{16}$$

Due to complementary slackness, the transmission shadow prices are zero if the constraint is not binding, positive if the upper constraint $\mathbf{f} \leq \overline{\mathbf{f}}$ is binding and negative if the lower constraint $\mathbf{f} \geq \underline{\mathbf{f}}$ is binding. If the upper flow limits are positive and the lower flow limits are negative, as is usually the case, then the congestion rent is non-negative, from (16).

The congestion rent can be expressed in yet another useful form, as the sum of the *profit* that each line makes under the so-called admittance pricing schemes [25], [26], [27], i.e., the sum of the product of line flows and price differentials between nodes. Starting with (12) and using (2),

$$\mathcal{K} = (\mathbf{l} - \mathbf{p})' \boldsymbol{\pi} \tag{17}$$

$$= \mathbf{f}' \mathbf{A} \boldsymbol{\pi} \tag{18}$$

$$= \sum_{\ell} f_{\ell} \left(\pi_{n_{\ell}} - \pi_{m_{\ell}} \right).$$
 (19)

Note that while the congestion cost are usually positive, as stated above, some lines may not be profitable. For these *unprofitable lines*, the flow is from a higher price node to a lower price node. As such, the flow on these lines are not economic, but occur exclusively because flows are dictated by Kirchoff's Laws rather than simple transportation laws.

IV. ALGORITHM STRUCTURE

As discussed in Section I, current MIP implementations for transmission topology control, while assuring good transmission topology solutions and providing significant cost savings, are prohibitively computationally intensive. This section presents a general algorithm structure for topology control heuristics. The objective of the heuristics is to obtain improved transmission topologies for production cost reduction with little computational effort. The algorithm is specified in Fig. 1. This simple algorithm structure sequentially solves an OPF and, given the OPF results, if the OPF is feasible, selects a set of candidate switches to change status (infeasibility is accounted for in the Step 3 check). Optimality is not guaranteed with this procedure, since there is no co-optimization of dispatch and transmission topology, in contrast to [15]-[19]. However, this structure has the potential to attain high quality switching and dispatch solutions with very low computational effort, as will be exemplified in the next sections.

There are three characterizing elements for each heuristic algorithm: i) the switching criteria for Step 1, ii) the switchable



Fig. 1. Flow chart describing general algorithm structure of economic transmission topology control heuristics.

set update criteria for Steps 3 and 4, and iii) the stopping criteria for Step 4. These criteria need to be specified.

The switching criteria can select one or more switches as candidates to change status, and open or close these switches. The selection can be made applying the same criteria in every iteration, or by applying criteria that are a function of the iteration number or of the results of previous iterations. The criteria itself can take a number of forms, including the simple use of OPF primal and dual solution variables for the selection, or be based on the result of optimization problems.

The switchable set update criteria determines which switches are allowed to change their states in the current iteration. This update has two objectives: improve cost performance and account for reliability requirements. A very simple reliability requirement could be, for example, that the number of lines connecting a certain node cannot be less than a specified number. Regarding cost performance, some heuristics may not change the status of a switch twice. Also, once a set of switching candidates is found to perform poorly, this same set is precluded from being used again in the next iteration but may be allowed to change status in later iterations, depending on the switchable set update criteria.

The stopping criteria determines the number of iterations applied, and can include a number of conditions. Clearly, the computations would stop if no lines meet the switching criteria, or if the switchable set is empty. Additionally, the algorithm may have a pre-set maximum number of iterations, and/or maximum number of status changes. Further, it may stop if cost reductions exceed a certain threshold, or if the computation time is above a pre-defined limit.

Section V specifies a heuristic algorithm by providing detailed rules for each one of these elements.

V. THE LINE PROFIT TRANSMISSION TOPOLOGY CONTROL HEURISTIC

The proposed *line profit* transmission topology control heuristic is named after the metric used in the switching

criteria. The characterizing elements of the heuristic are in the following subsections.

A. Switching Criteria

In each iteration, the line profits, i.e., $f_{\ell}(\pi_{n_{\ell}} - \pi_{m_{\ell}})$, are computed. The most unprofitable line, if any, is selected as a candidate for opening. Disconnecting any such line removes a negative term from the summation in (19), thereby increasing it. Considering that the optimal cost C resulting from the solution of (7)-(10) can be expressed as the sum of the load payments minus the generation gross margin minus the congestion rent,

$$C = \pi' \mathbf{l} - (\pi - \mathbf{c})' \mathbf{p} - \mathcal{K}, \qquad (20)$$

removing a negative term in \mathcal{K} appears to reduce the production costs, at least in an incremental sense. Next we show that this is the case using two well-known sensitivities.

The power transfer distribution factor ϕ_{ℓ}^{mn} , or PTDF, gives the sensitivity of the flow on line ℓ with respect to a power transfer from node m to node n. The PTDF ϕ_{ℓ}^{mn} can be expressed in terms of shift factors as [23]

$$\phi_\ell^{mn} = \psi_\ell^m - \psi_\ell^n. \tag{21}$$

The term in (19) corresponding to line k satisfies

$$f_k \left(\pi_{n_k} - \pi_{m_k} \right) = -f_k \mathbf{A}_k \left(\lambda \mathbf{1} + \mathbf{\Psi}' (\overline{\boldsymbol{\mu}} - \boldsymbol{\mu}) \right) \quad (22)$$

$$= -f_k \mathbf{A}_k \mathbf{\Psi}' (\overline{\boldsymbol{\mu}} - \underline{\boldsymbol{\mu}})$$
(23)

$$= -f_k \phi^{n_k m_k'} (\overline{\mu} - \underline{\mu})$$
(24)
$$= f_k \phi^{m_k n_k'} (\overline{\mu} - \underline{\mu})$$
(25)

$$= f_k \sum \phi_{\ell}^{m_k n_k} (\overline{\mu}_{\ell} - \mu_{\ell}). \quad (26)$$

$$= f_k \sum_{\ell} \phi_{\ell}^{max} (\mu_{\ell} - \underline{\mu}_{\ell}), \qquad (26)$$

where $\phi^{m_k n_k}$ is the column vector of PTDF of all lines for transactions from node m_k to n_k , i.e., for transactions in the direction of line k, and A_k is the row in A corresponding to line k. Thus, the profit each line makes is equal to the shadow cost impacts a transaction in the direction of the line would make on each binding constraint, multiplied by the line flow.

The *line outage distribution factor* o_{ℓ}^k , or *LODF*, gives the sensitivity of the flow on line ℓ with respect to a reduction in the flow on line k. The LODF o_{ℓ}^k is given by [23]

$$o_k^k = -1, (27)$$

$$o_{\ell}^{k} = \frac{\phi_{\ell}^{m_{k}n_{k}}}{1 - \phi_{k}^{m_{k}n_{k}}}, \ell \neq k,$$

$$(28)$$

for $\phi_k^{m_k n_k} \neq 1$, and is 0 for all $\ell \neq k$ if $\phi_k^{m_k n_k} = 1$. The PTDF $\phi_k^{m_k n_k}$ of line k for transactions from its *from* node to its *to* node is positive and between 0 and 1, so that

$$1 - \phi_k^{m_k n_k} \ge 0. \tag{29}$$

The linear estimate of economic effects due to a line outage is

$$f_k \sum_{\ell} o_{\ell}^k (\overline{\mu}_{\ell} - \underline{\mu}_{\ell}) = f_k \Big(-(\overline{\mu}_k - \underline{\mu}_k) + \sum_{\ell \neq k} \frac{\phi_{\ell}^{m_k n_k}}{1 - \phi_k^{m_k n_k}} (\overline{\mu}_{\ell} - \underline{\mu}_{\ell}) \Big). (30)$$

TABLE I SIMULATION RESULTS SUMMARY: LINE PROFIT HEURISTIC PERFORMANCE

metric	initial topology	unconstrained	line profit heuristic			
			unlimited i	limited i	reduced set	pre-ordered
expected cost (\$)	$129,598 \pm 3550$	$117,045 \pm 3168$	$120,926 \pm 3243$	$121,174 \pm 3250$	$124,045 \pm 3360$	$120,623 \pm 3233$
expected savings (%)	n/a	9.49 ± 0.38	6.51 ± 0.20	6.29 ± 0.20	4.17 ± 0.18	6.73 ± 0.24
min / max savings (%)	n/a	$0.99 \ / \ 19.17$	0.68 / 11.18	$0.57 \ / \ 10.60$	$0.52 \ / \ 9.07$	$0.58 \ / \ 10.94$
median / max iterations	n/a	n/a	41 / 68	15 / 15	33 / 47	15 / 15
lines disconnected (median)	n/a	n/a	16	8	17	9
expected computation time (s)			0.67	0.29	0.73	0.31

For non-binding lines, $(\overline{\mu}_k - \underline{\mu}_k) = 0$, therefore

$$f_k \sum_{\ell} o_{\ell}^k (\overline{\mu}_{\ell} - \underline{\mu}_{\ell}) = \frac{f_k (\pi_{n_k} - \pi_{m_k})}{1 - \phi_k^{m_k n_k}}.$$
 (31)

For all binding unprofitable lines, from (30), (31) and using $-1 \le \|\phi_k^{m_k n_k}\|,$

$$f_k \sum_{\ell} o_{\ell}^k (\overline{\mu}_{\ell} - \underline{\mu}_{\ell}) \le f_k (\pi_{n_k} - \pi_{m_k}) < 0.$$
 (32)

As such, all unprofitable lines are lines whose outage would lead to reduced production costs in an incremental sense.

Note that the line profit heuristic only opens closed switches, it does not close open switches since line profits cannot be computed for disconnected lines. Since flows and prices are readily available from the solution of the DCOPF, the switching criteria entail a negligible computational effort, which provides a significant advantage when compared to other methods in the literature.

B. Switchable Set Update Criteria

The switchable set is reduced in each iteration by removing from it the most unprofitable switchable line, regardless of whether removing the line leads to cost savings or not. Once a line is removed from the switchable set, it is not reinstated for the rest of the algorithm.

The switchable set update criteria also enforces a relaxed form of the "n-1" reliability requirement, as follows: at no point a load or generator bus is allowed to be served by less than two lines. Thus, if in any iteration such a bus becomes connected to the rest of the system by only two lines, then these two lines are removed from the switchable set.

C. Stopping Criteria

Two stopping criteria are used. The "unlimited iterations" criterion makes the heuristic stop searching when no switchable unprofitable lines are found. The "limited iterations" criterion has an iteration limit *I*.

VI. SIMULATION RESULTS

The line profit heuristic was tested on the IEEE 118-bus test system. This test system represents a portion of the AEP network (in the Midwestern US) as of December, 1962. The version of the test system employed is available at [28]. It consists of 118 buses, 54 generators, and 194 branches, all of which are connected. The load is 3,668 MW. The generation economic characteristics developed for this test system are detailed in the Appendix.

To study the performance of the line profit heuristic under different system conditions, we maintain a fixed load and perform a Monte Carlo simulation where the fuel costs and the available wind generation are randomly varied. Fuel costs are assumed to meet the condition that the cost of coal is lower than the cost of natural gas which in turn is lower than the cost of fuel oil. The coal price sample is taken from a uniform distribution between 0.5 and 3 \$/MBTU. The natural gas price is drawn from a uniform distribution with an upper limit of \$10/MBTU and a lower limit of the maximum of the coal price and \$2/MBTU. The fuel oil price is drawn from a uniform distribution with upper limit \$12/MBTU and a lower limit of the maximum of the natural gas price and \$5/MBTU. Wind units are dispatched at \$0/MWh. Available capacity of the wind power units is assumed to be uniformly distributed between 0 and their rated capacity. The available wind power of units within each of the following sets of buses is assumed to be perfectly correlated: {10, 31}, {46, 70}, and {87, 103, 111}. The cross-correlation coefficient between these sets is assumed to be 0.75, to model the effects of geographic proximity.

The sample size used for the Monte Carlo simulation is 100. Two benchmark cases are used to evaluate the performance of the heuristic: the case with initial topology and the unconstrained case. The DCOPF with the initial topology provides an upper bound for the production costs, and gives the reference against which production cost savings are measured, while the unconstrained DCOPF provides a lower bound for production cost and an upper bound for production cost savings with transmission topology control. The results for these two benchmark cases are in the first two columns of Table I. Note that the congestion costs, i.e., the maximum attainable savings, are 9.49% of the production costs with the initial topology. The initial transmission topology has between 1 and 4 lines binding for the samples taken, with an average of 2.3 binding lines. Only 7 lines had limits binding in at least one of the 100 samples with the initial topology. Thus, while there are variations in the congestion patterns due to changes in fuel costs and renewable generation, these variations are moderate.

We implemented the line profit heuristic in Matlab, using PowerWorld 15 as the DCOPF solver. The average production



Fig. 2. Initial dispatch around line $25 \rightarrow 23$, one of the lines disconnected most frequently. This line feeds the binding constraint $23 \rightarrow 32$.

costs reductions obtained were 6.51%, as detailed in the third column of Table I. Thus, about 68.6% of the maximum attainable savings were obtained with this heuristic. Note that these maximum attainable savings are for the case with no congestion. The savings magnitude leads us to suspect that the transmission topologies attained with the line profit heuristic are probably close, cost-wise, to the optimal topologies.

To gain insight on transmission topology control and how the line profit heuristic works, let us consider the decision to outage two of the most frequently disconnected lines. The initial topology and dispatch focusing on line $25 \rightarrow 23$ are shown in Fig. 2. Buses 25 and 26 have large natural gas-fired generators that are fully dispatched. Power from these units feed the binding line $23 \rightarrow 32$ through line $25 \rightarrow 23$. Line $25 \rightarrow 23$ is highly unprofitable, because it takes a significant amount of flow (237 MW in the sample shown) from higher price node 25 (\$53/MWh) to lower cost node 23 (\$37/MWh in the sample shown). Opening this line in the sample considered results in line $23 \rightarrow 32$ having excess capacity, while line $26 \rightarrow 30$, as well as another line downstream of it, become binding. The savings obtained by opening this line are mainly due to an increase in low cost generation at bus 89 (quite far from bus 23, downstream of bus 24), and a reduction in higher cost generation, including, counter to intuition, the plant at bus 26 which becomes a marginal unit.

The second most disconnected item turned out to be transformer $65 \rightarrow 66$. Let us consider the realized initial topology and dispatch shown in Fig. 3. A fully dispatched coal unit is connected to bus 69, while a marginal gas-fired unit is connected to bus 65 (this unit controls the enforcement of binding constraint $68 \rightarrow 65$). Also, a marginal coal-fired unit is downstream of bus 77, in bus 89. The binding constraint $68 \rightarrow$ 65 feeds the branches from bus 65 to buses 58, 66 and 64, all of which are unprofitable, since they transfer flow from a binding constraint to widely connected nodes. Transformer $65 \rightarrow 66$ tends to be the least profitable. Opening this transformer allows the replacement of expensive generation from marginal unit 65 with lower cost generation from marginal unit 89. Note



Fig. 3. Initial dispatch around transformer $65 \rightarrow 66$, one of the branches disconnected most frequently. This transformer is fed by the binding constraint $68 \rightarrow 65$.



Fig. 4. Average production costs as a function of the iteration number.

the marked differences between this line branch disconnection and the previous one. The transformer disconnected is fed by a binding constraint (rather than feeds the binding constraint), and does not change the set of marginal units or the binding constraints (the previous line changed both).

Fig. 4 shows the reduction in average costs as a function of the iteration, normalized so that 1 refers to the average cost with the initial topology, and 0 refers to the average cost with no transmission congestion. As found in [15], most of the cost reductions are realized in the first few iterations, and brought by the change of status of a few switches.

The heuristic performance when at most 15 iterations are allowed is shown in Table I in the "limited *i*" column. The average production costs reductions were 6.29%. Hence, about 96.6% of the heuristic savings were obtained in the first 15 iterations. Fig. 5 shows, for each line, the number of samples (out of the 100 samples) in which the line was disconnected, with the lines reordered by decreasing number of switchings. Note that six lines were switched in at least 74% of the samples. These lines tended to be switched in the first iterations, and bring the most savings. Also, note that only 16 switches are operated in more than 10% of the samples (if the iterations are limited to 15). These results indicate that only a relatively small portion of system switches would attain the majority of production cost savings. Also, if there is a need to upgrade switches or their controls for frequent operation,



Fig. 5. Number of switching operations for the most switched lines; iterations limited to 15.

perhaps only a few switches are worthwhile upgrading for inclusion in the control algorithm.

If the six lines that bring the most savings were not allowed to change status, the heuristic would still results 4.17% in savings, as indicated in column "reduced set" of Table I. These represent over 43% of the total available savings. Thus, while it is affected, the performance of the heuristic does not fully depend on being able to switch these six most-effective lines.

To analyze the path-dependency of the heuristic, instead of selecting the most unprofitable line as a candidate for switching, we pre-order the lines. For each iteration in the 100 samples, the most frequently switched line is selected as the switching candidate for that iteration. If the line was already selected as the candidate for a previous iteration, then the most frequently switched line that was not chosen for a previous iteration is selected. The results, for up-to 15 iterations, is shown in the column labeled "pre-ordered" in Table I. Note that due to the improved ordering of the candidate lines, the savings, 6.74%, are higher than those obtained with the unlimited iterations and not pre-ordered heuristic, and results in an increase of 7% over those attained with the same number of iteration and without the pre-ordering. Thus, we conclude that for this test system the performance of the heuristic is path-dependent and benefits from an improved ordering of candidate lines, as one would suspect from the results in [15]. However, the improvements from the pre-ordering were not very high.

The total computation time employed per sample averaged 0.67 seconds, on a Dell Latitude D620 laptop with a 1.83 GHz Intel Centrino Duo processor and 2 GB of RAM. When the iterations were limited to 15, the average computation time was reduced to 0.29 seconds. Comparing the performance and computational cost of this heuristic with any of the implementations presented in the literature, we note that the savings attained are similar, while the computational cost is reduced by 3 to 4 orders of magnitude. The significance of such impressive computational effort savings is that with these short computation times, economic transmission topology control can be analyzed for realistic systems, as well as practically included in computationally intensive problems that otherwise may be prohibitively slow, such as unit commitment and production simulation.

VII. CONCLUDING REMARKS

This paper has presented an algorithm structure for fast transmission topology control heuristics, and the application of the structure in the design of the "line profit" heuristic. The heuristic disconnects the single line that is the most unprofitable, which we show to reduce production costs in an incremental sense. Simulation results on an implementation of the IEEE 118-bus test system shows that the heuristic can be very effective in lowering production costs, reducing the initial congestion costs by 2/3 in this case. As the required information to determine the line profit is readily available from the OPF solution, the computational effort is very small, with computational times up to four orders of magnitude lower than the approaches in the literature.

Future work will take several directions, including the design and analysis of other fast heuristics, the study of transmission topology control impacts on nodal prices and ancillary services needs, with the corresponding markets design implications, the effectiveness of transmission control heuristic when applied to realistic, large-scale test systems, and the inclusion of transmission topology control on complex problems, such as unit commitment and operations planning.

APPENDIX

IEEE 118-BUS TEST SYSTEM ECONOMIC MODEL SPECIFICATION

The IEEE 118-bus test system [28] consists of a power flow with line limits but with no generation or load data beyond a fixed dispatch. This section specifies the economic model developed for the test system. The power flow has 54 generators. The 15 units negative real power dispatch are assumed to be part of an end-user facility with self-generation, and as such, are not assigned an economic model and their dispatch is maintained. Units with zero dispatch in the original power flow are assumed to be residual fuel oil-fired internal combustion plants, with 10,000 BTU/kWh efficiency, 0 MW min capacity and 20 MW maximum capacity. These units are in buses 15, 18, 19, 32, 34, 36, 55, 56, 62, 74, 76, 77, 82, 85, 92, 104, 105 and 110. The remaining units are assigned a unit type, fuel type, efficiency and minimum and maximum capacities based loosely on their dispatch in the original power flow. Fixed operation costs, as well as variable operation and maintenance costs, are set to 0. The economic model is shown in Table II. In the third column of this table, ST stands for steam turbine, CC stands for combined cycle, JE stands for jet engine, and WT stands for wind turbine.

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 TABLE II

 IEEE 118-BUS GENERATION ECONOMIC MODEL

bus	fuel	type	heat rate	capacity
			(BTU/kWh)	max/min (MW)
49	coal	ST	9,600	300 / 150
69, 80	coal	ST	10,000	550 / 192.5
89	coal	ST	10,200	650 / 227.5
25, 100	natural gas	CC	7,000	300 / 150
26	natural gas	CC	7,000	500 / 250
12	natural gas	GT	12,000	200 / 67.5
59, 61	natural gas	GT	10,800	160 / 120
6	natural gas	JE	9,500	200 / 37.5
54	natural gas	JE	9,500	50 / 37.5
65	natural gas	ST	9,800	450 / 157.5
66	natural gas	ST	10,000	450 / 157.5
10	wind	WT	n/a	450 / 0
31	wind	WT	n/a	20 / 0
46, 70, 87, 111	wind	WT	n/a	100 / 0
103	wind	WT	n/a	200 / 0

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