

Fabricating and Measuring the Resonant Frequency of Oscillators using Analog Signal Processing

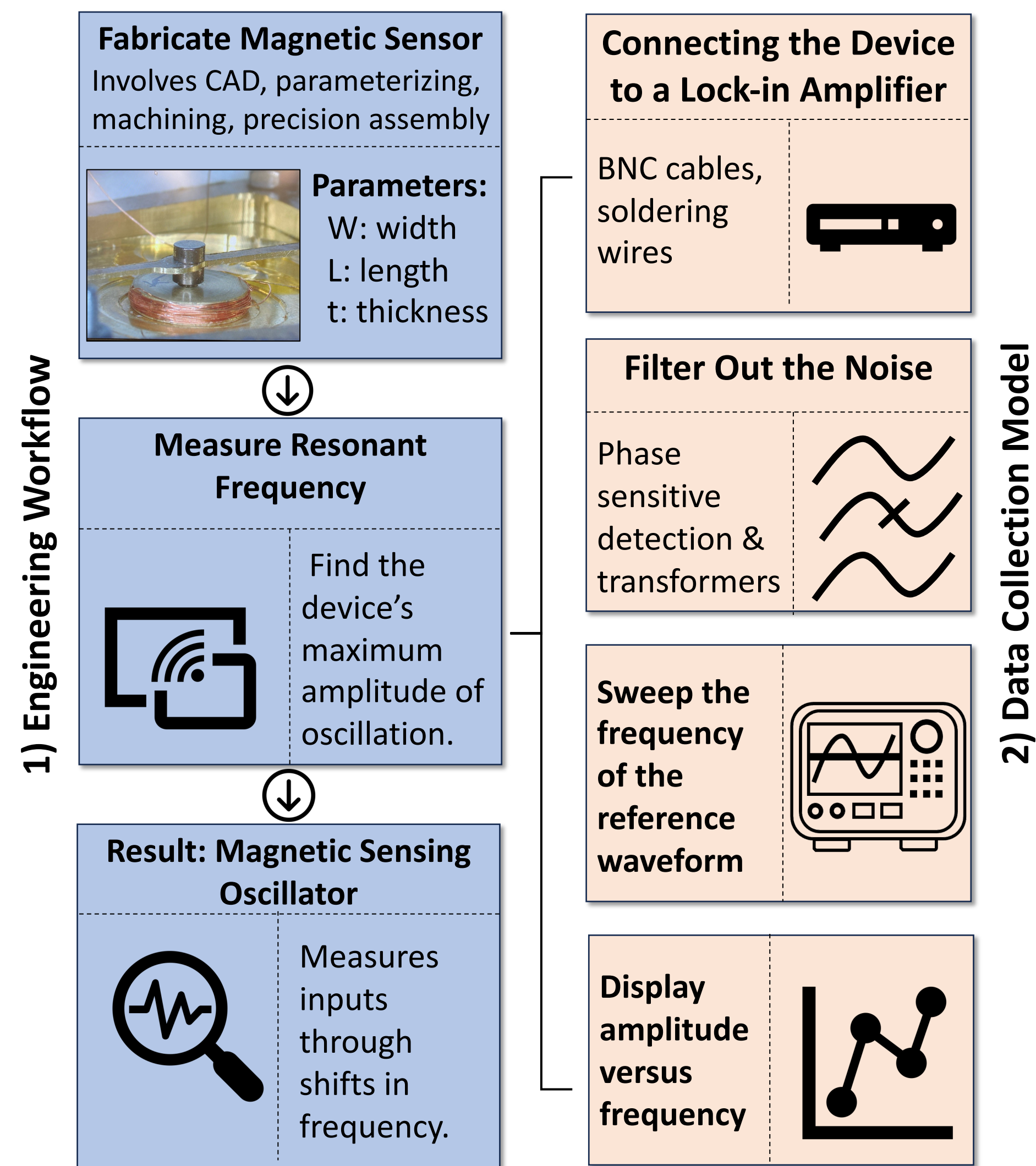
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VISUAL ABSTRACT



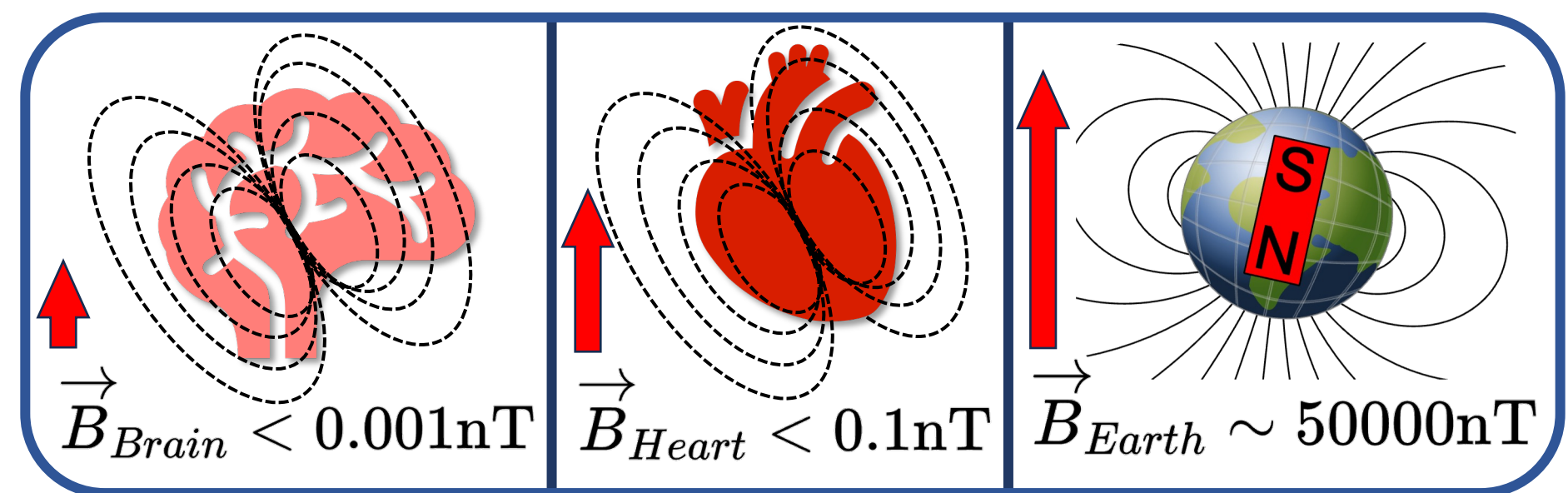
INTRODUCTION

PRECISION METROLOGY

Science of measurement focused on the accuracy and repeatability of measurements at a delicate level.

Magnetometer application in biomedical settings (e.g. cardiac imaging via motion of charged ions in the heart with magnetic field of ~100pT).

THE ENGINEERING PROBLEM: Methods of measuring signals at this scale, (e.g. **atomic magnetometers** & superconducting quantum interference devices, **SQUIDS**) are **expensive** and **inaccessible**.



Coupled Oscillator

Transduces input into a shift in **oscillation frequency**: essential for **high sensitivity measurement** (i.e., Gain) (Bouche 2024).

$$G = -\frac{1}{4\pi} \frac{F_C''(d_{eq})}{\Delta f_{min} k^{3/2} m^{1/2}} \frac{1}{(1 + F_C'(d_{eq})/k)^{3/2}}$$

Allows for **noise reduction tactics** such as **phase locking** & **signal amplification**. Noise: mechanical, electrical, geomagnetic.

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Key Equation: Resonance frequency (f) modeling the mechanics and signal the oscillator gives while utilizing Hooke's law.

Coupled oscillators could be a **novel approach** for measurements down to hundreds of zeptonewtons (10^{-21} N).

RESEARCH OBJECTIVES

RESEARCH QUESTION: Can a coupled oscillator be fabricated with the sensitivity to **pickup small signals** using **magnetic sensing**?

GOAL: Work towards **proof of concept** of a coupled oscillator for sensitive metrology.

Engineering Design Criteria for Oscillator

LOW COST

Minimal material needed compared to status quo.

PRECISION

Picks up small inputs such as micro Tesla.

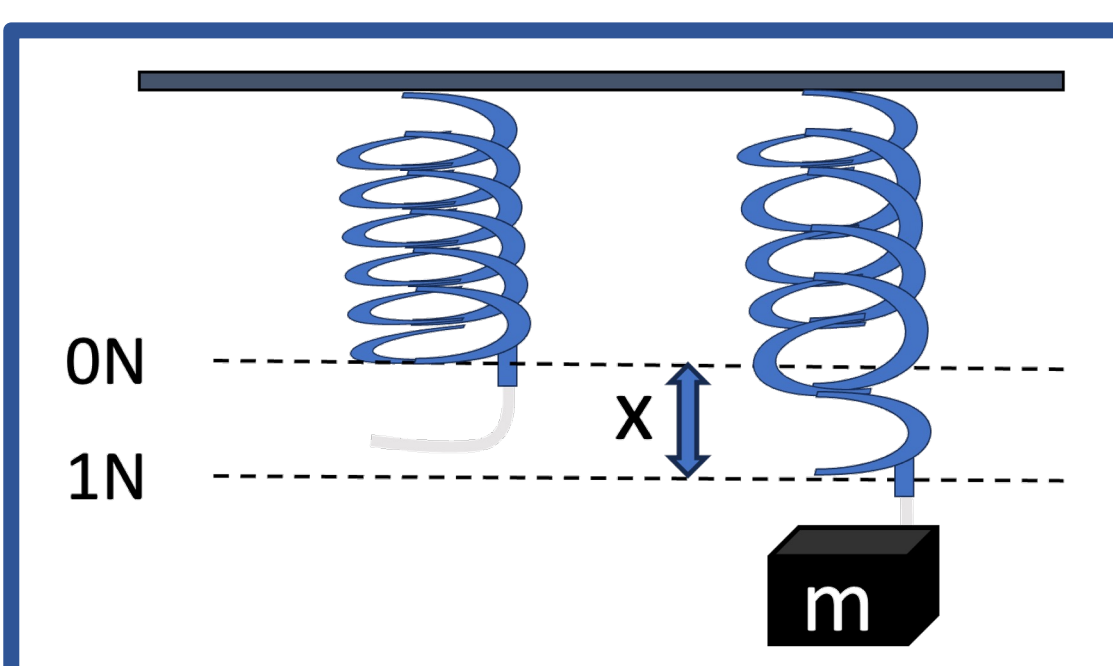
ROBUSTNESS

Atmospheric operation and has long lifespan.

WHERE TO START?: Design a resonator and establish parameters on paper than utilize equations such as...

a) Hooke's law:

$$\frac{F}{x} = k$$



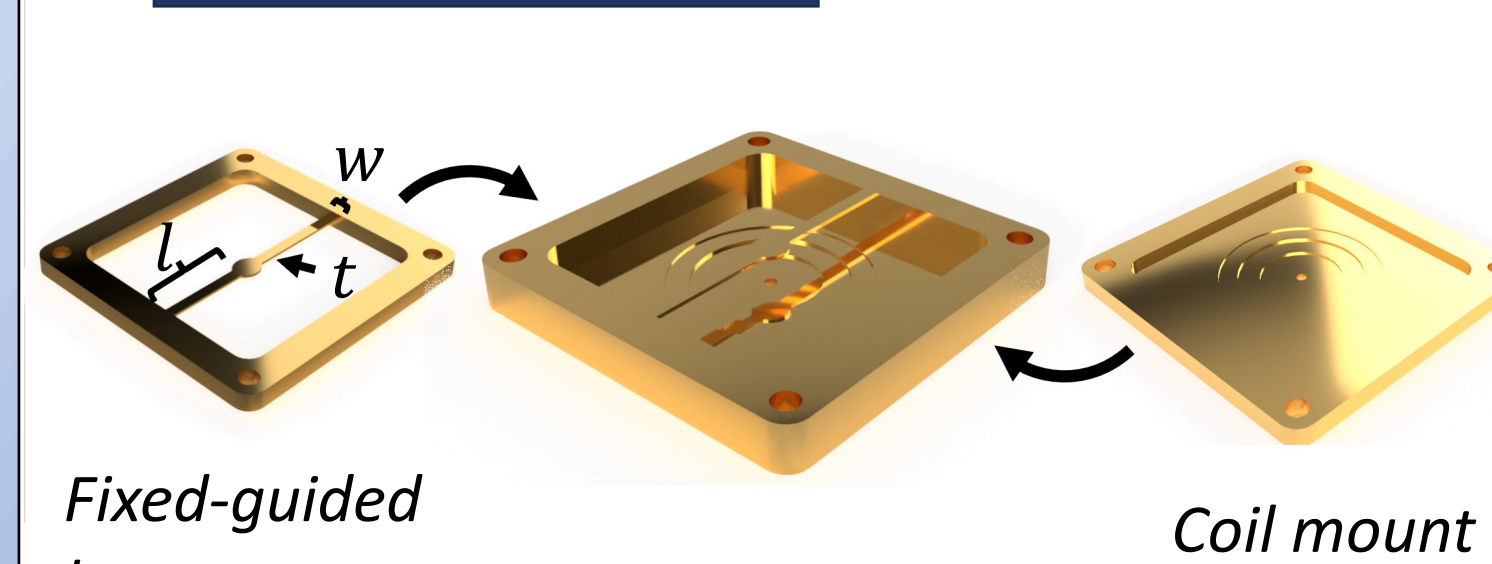
b) Youngs Modulus

$$\sigma = E\varepsilon$$

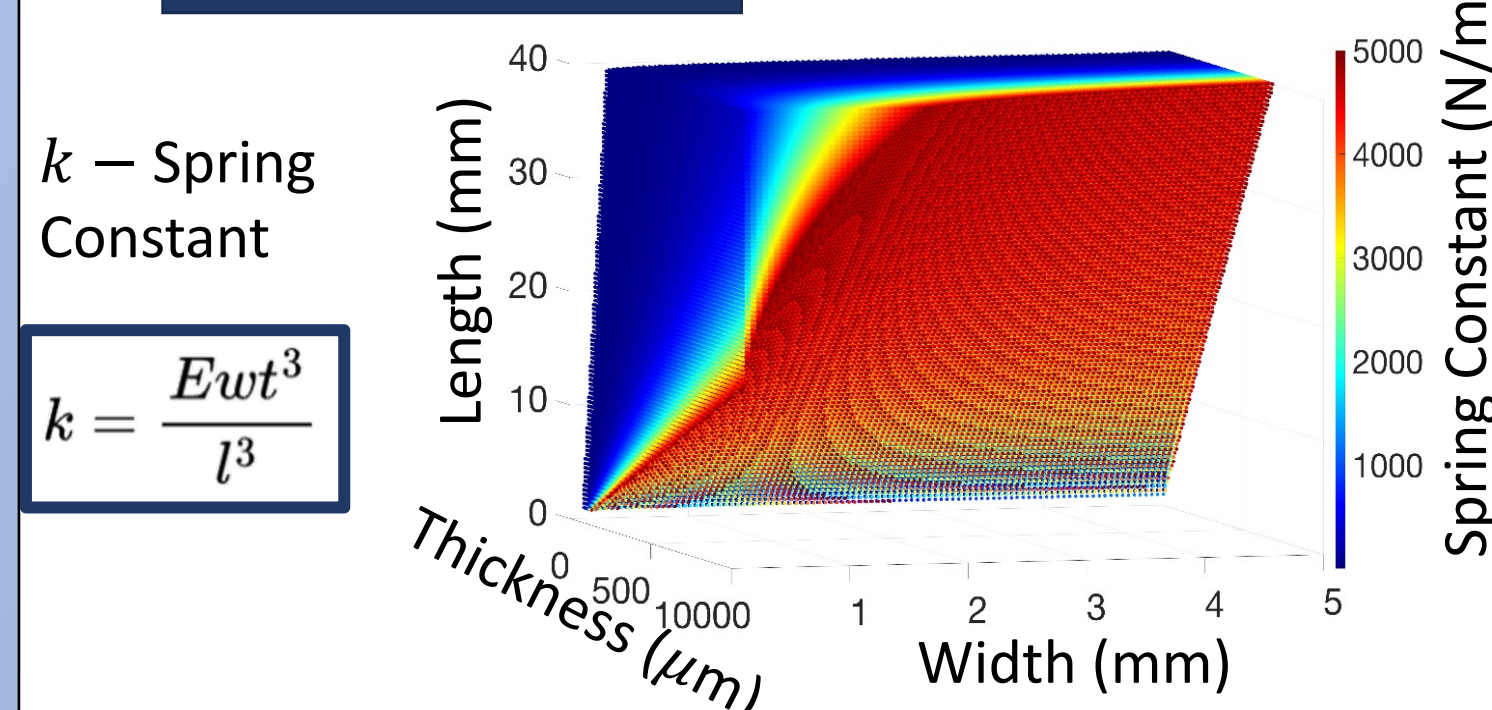
c) Plug into resonant frequency (f) & approximate the result in **MATLAB** and **COMSOL**.

1. DESIGN

1a) CAD Oscillator Design



1b) Setting Parameters

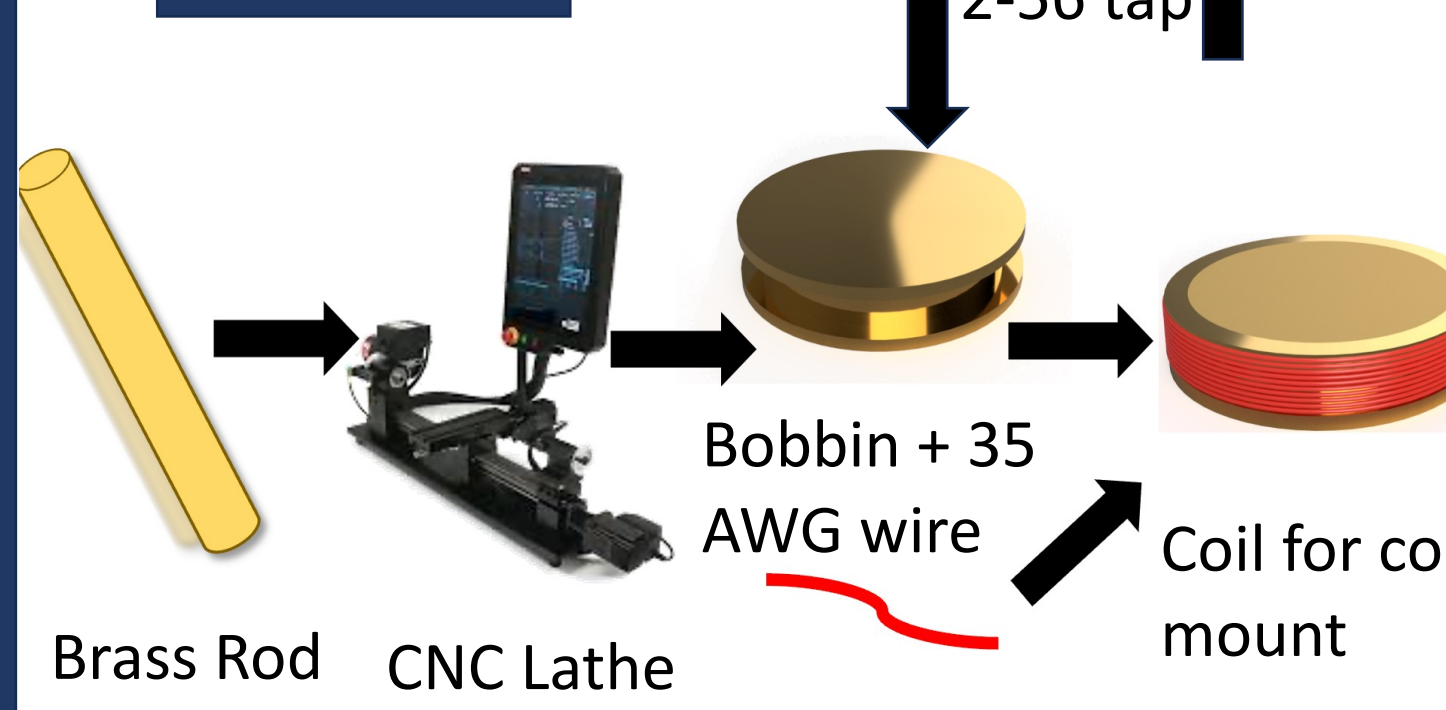


2. MANUFACTURE

2a) Milling Pieces For Assembly

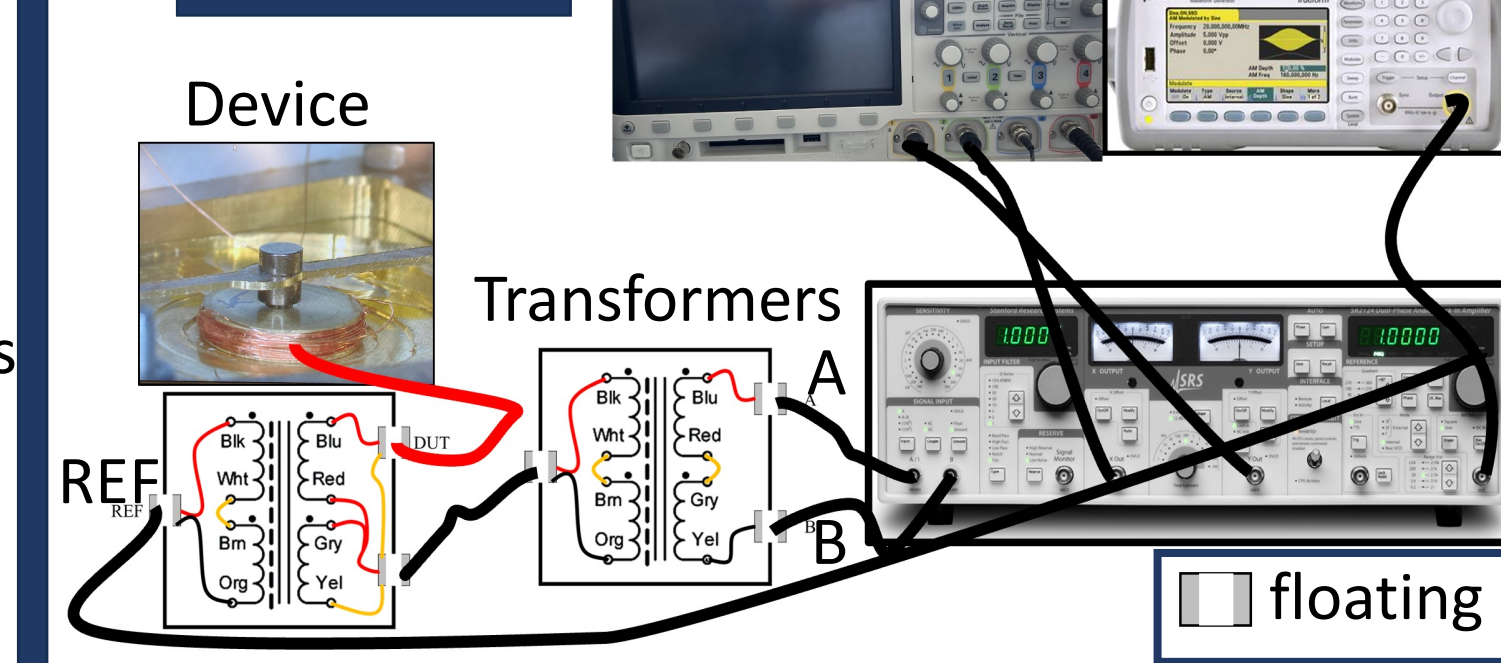


2b) Making a Coil

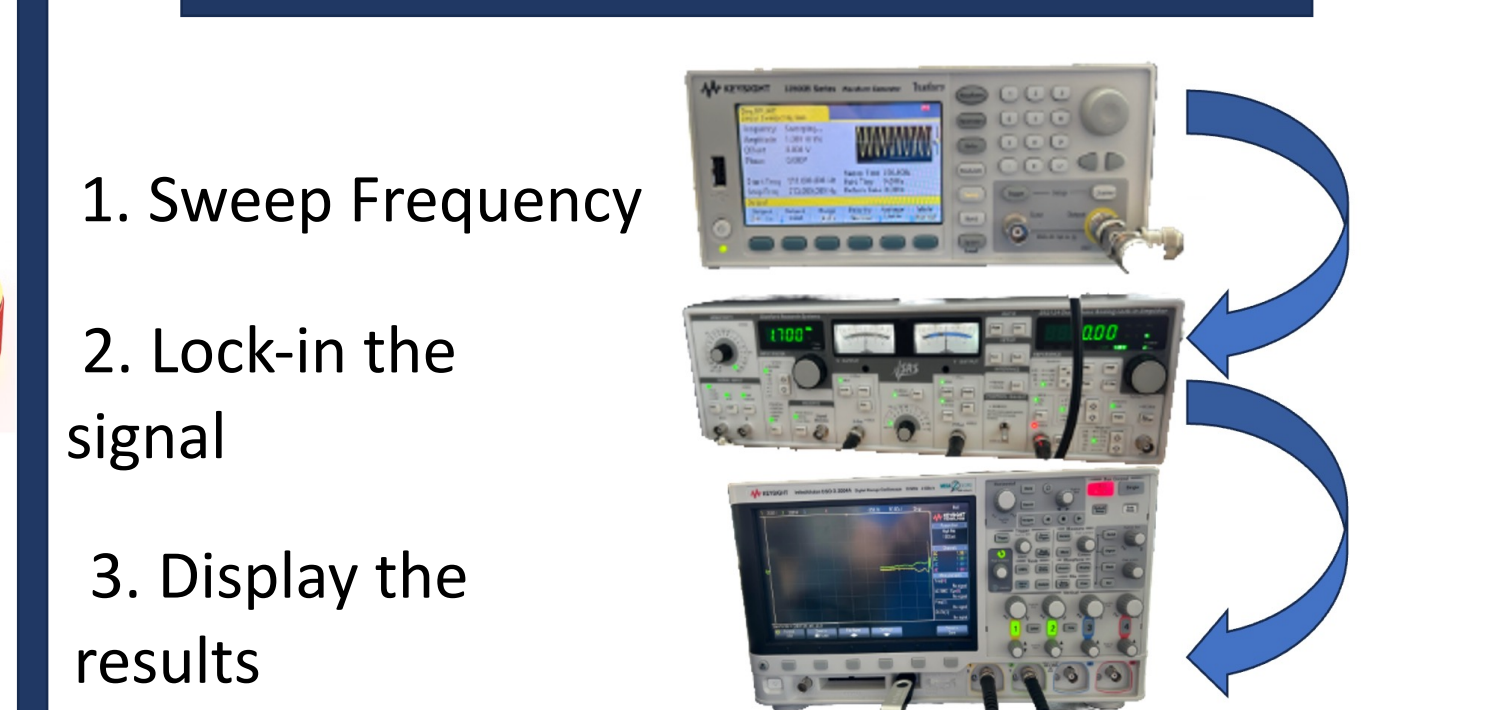


3. MEASURING

3a) Circuit Setup



3b) Sweep, Filter, Detect, & Display Data



RESULTS

Fig 1. Cut quality study of differing feed rates (mm/min)

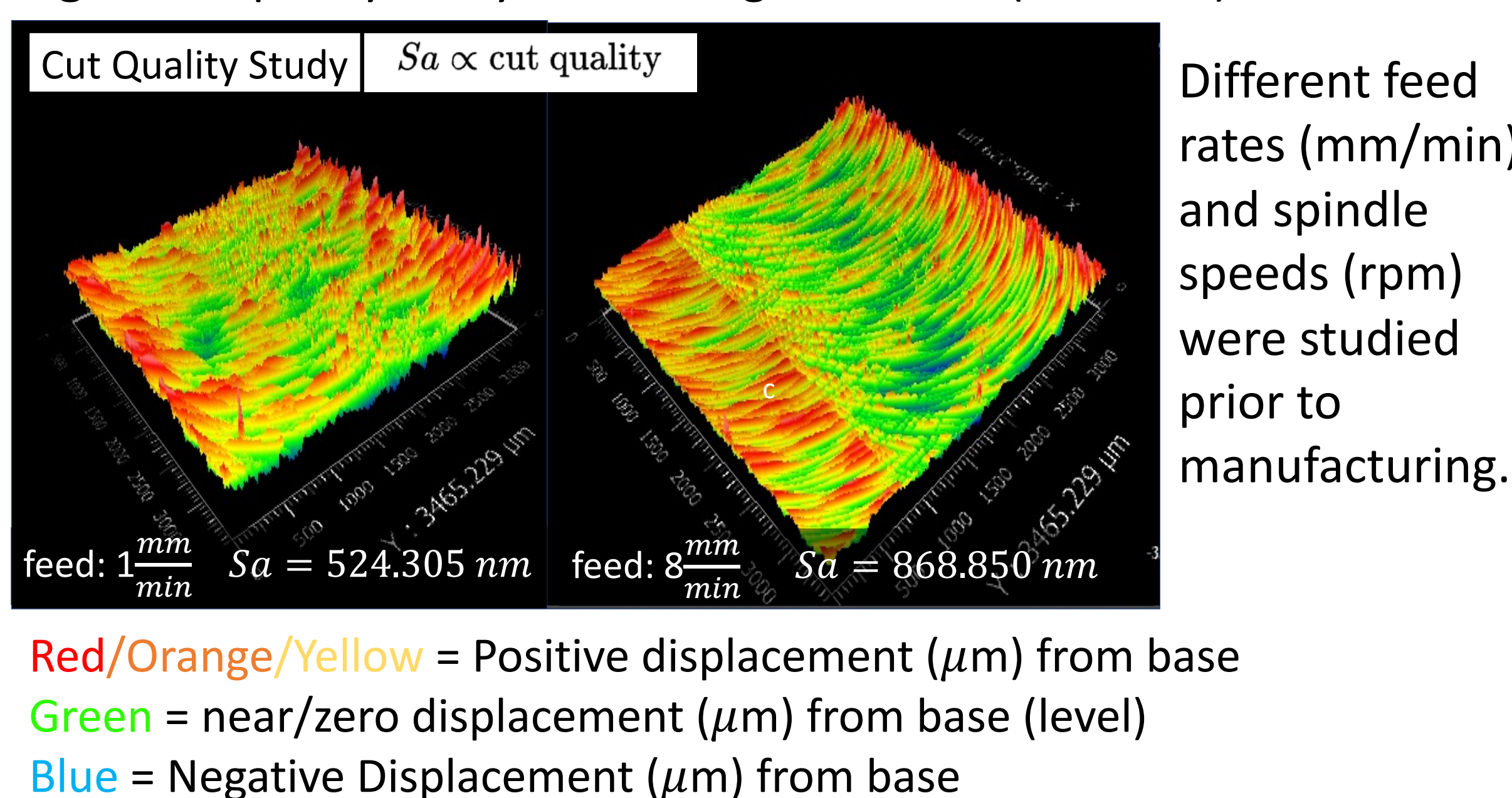


Fig 2. Bode plot of device in a turbomolecular pump ($<1.5e-5$ torr)

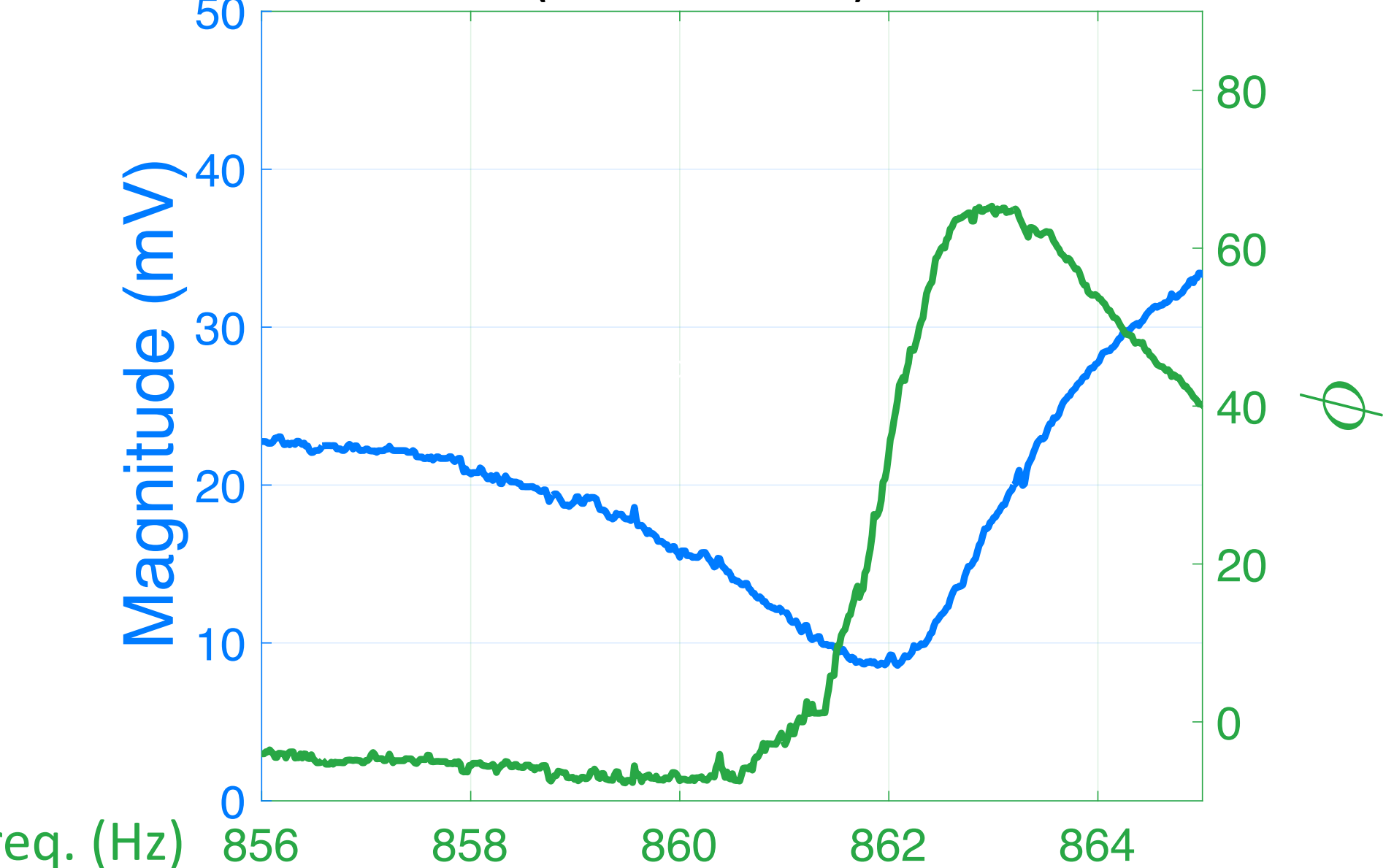


Fig 3. Bode plots of different amounts of magnets (bell jar)

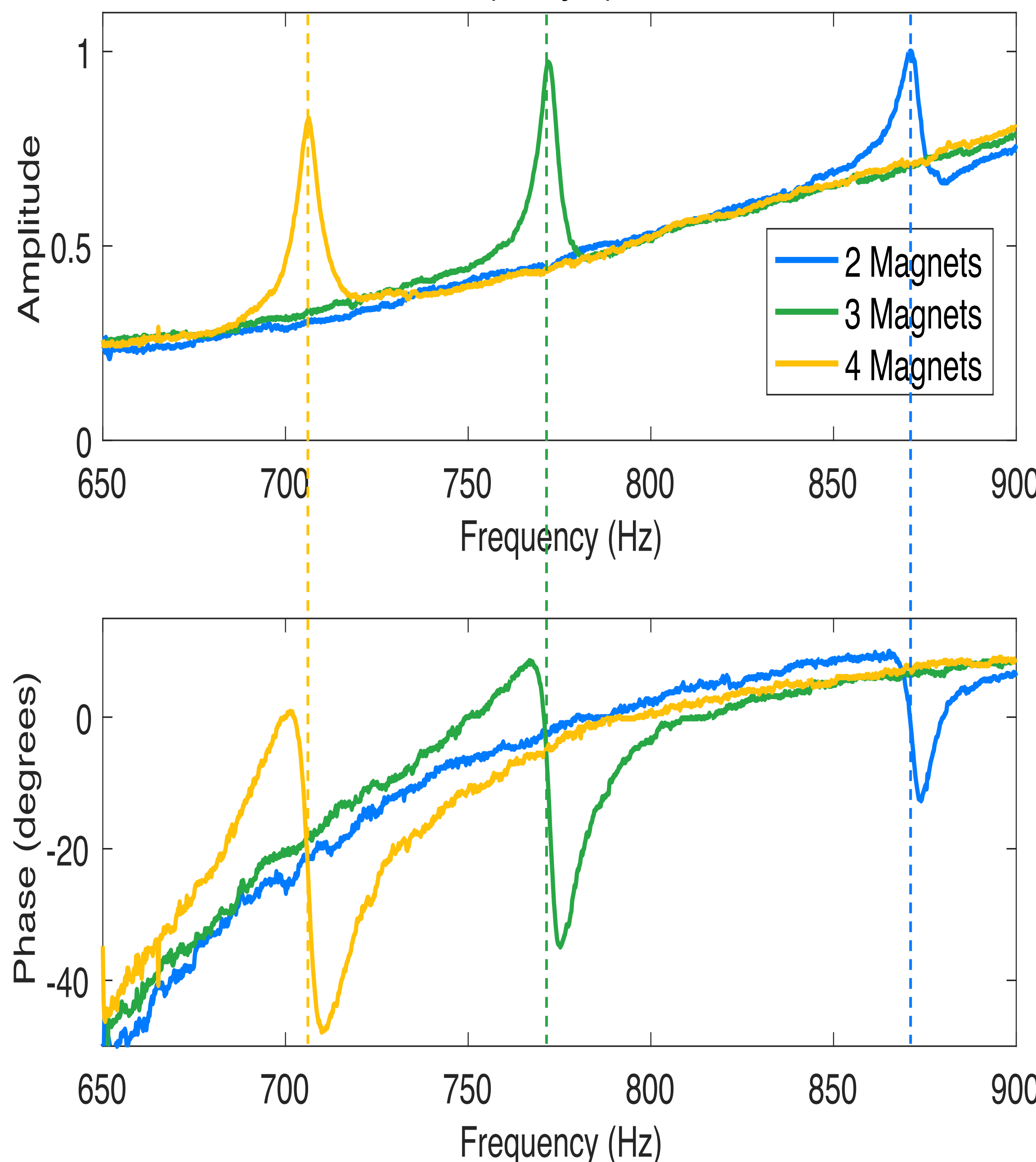


Fig 4. COMSOL simulated frequencies of the main mode with different magnets

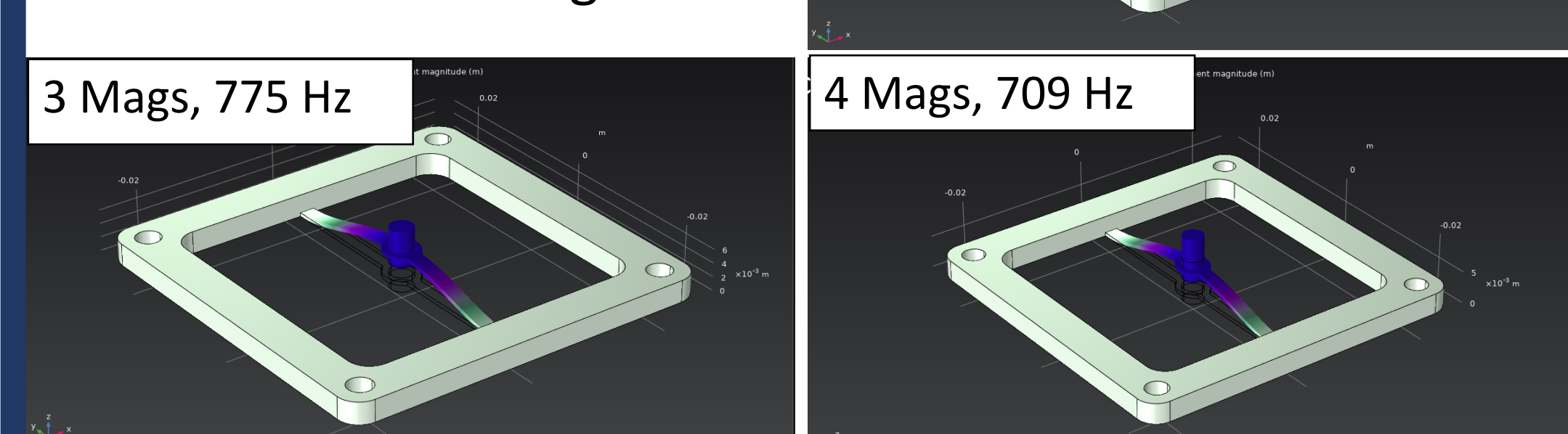
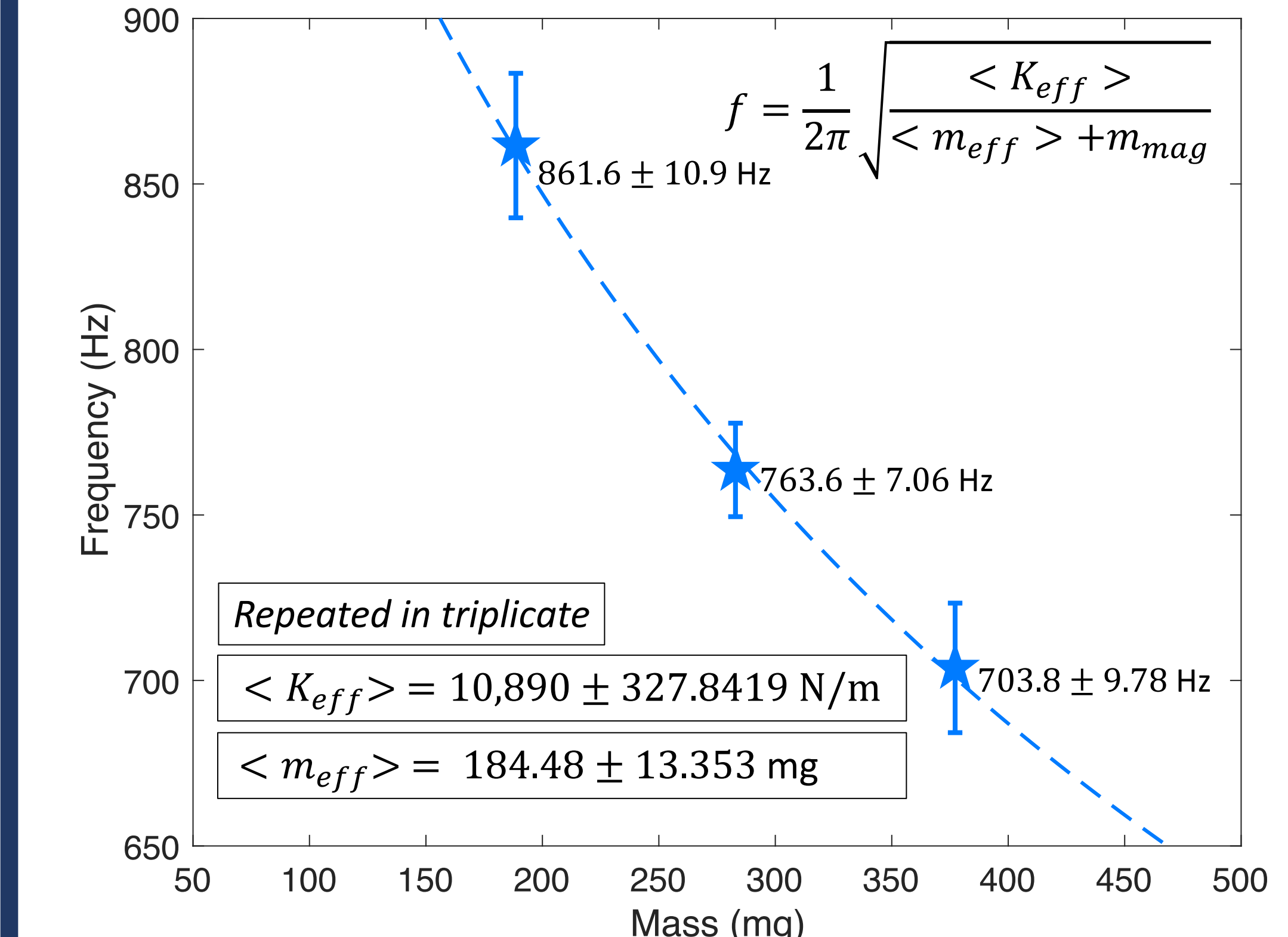
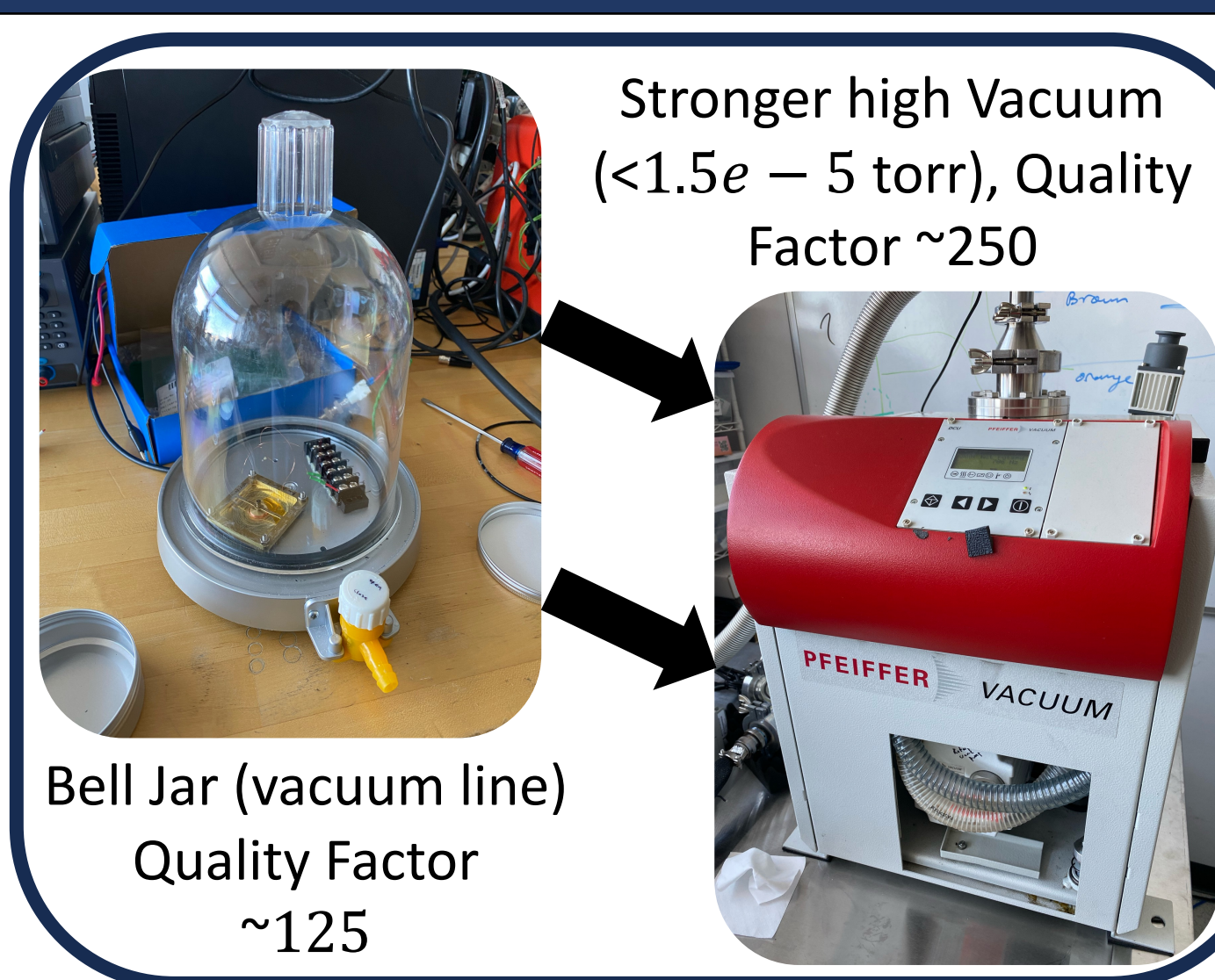


Fig 5. Statistical analysis of differing masses in a bell jar (~ 7.5 Torr)



CONCLUSIONS

- ❖ Slower feed rates lowers roughness (feed rate $\propto Sa$).
- ❖ This tested the concept of a magnetometer, next step is coupling this oscillator to applied magnetic field.
- ❖ A higher driving amplitude enhances **signal-to-noise ratio** of measurement.
- ❖ Back calculation of mass and spring constant were repeatable within error of 7% and 3% respectively (Fig. 5).



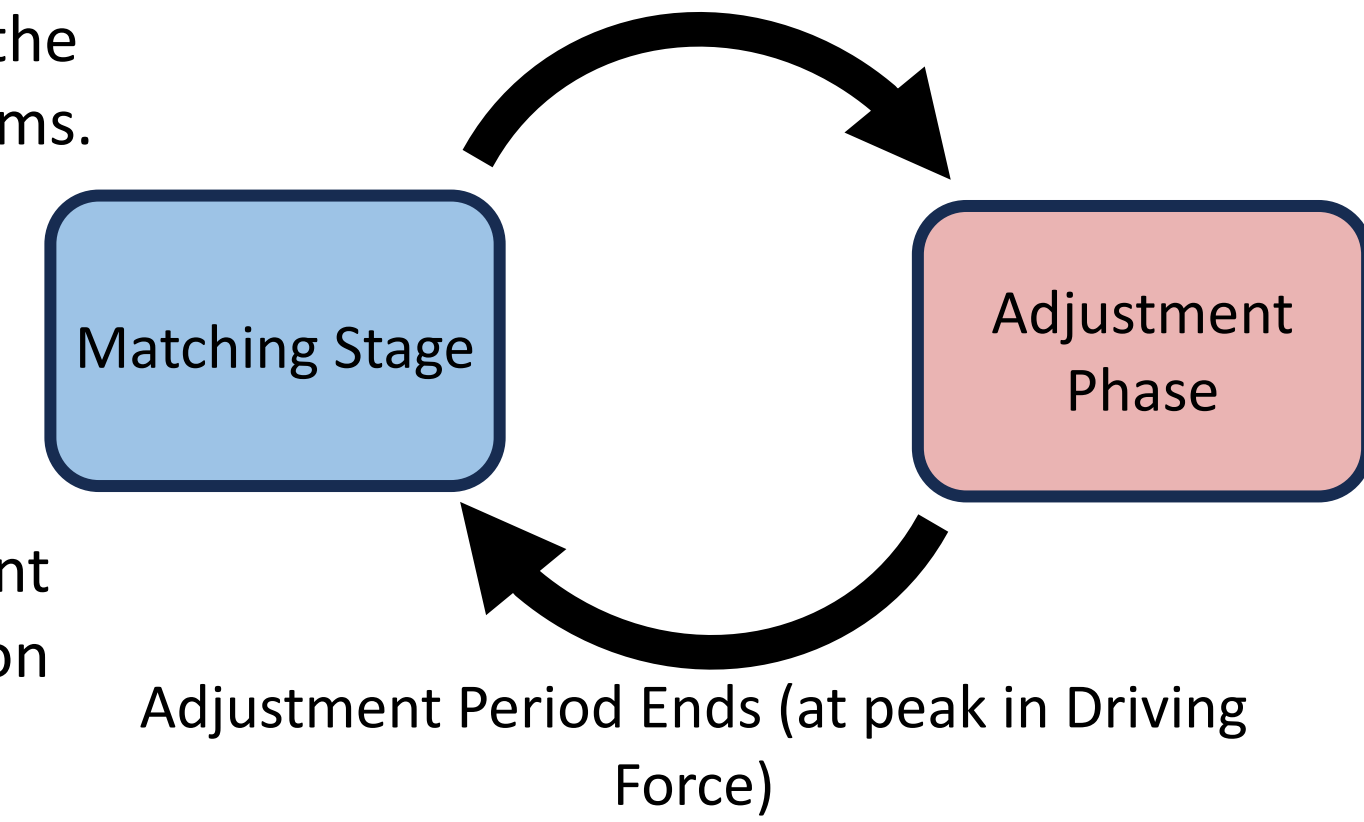
Stronger high Vacuum ($<1.5e-5$ torr), Quality Factor ~ 250

Allows for a **phase-locked loop** to track resonant frequency changes.

Detect shift in resonant frequency as a function of applied magnetic field.

Phase-Locked Loop Split in 2 States

Oscillator Reaches Peak Motion



KEY REFERENCES

- [1] Ian Bouche, Josh Javor, Abhishek Som, David K. Campbell, David J. Bishop; Zeptonewton and attotesla per centimeter metrology with coupled oscillators. *Chaos* 1 July 2024; 34 (7): 073133. <https://doi.org/10.1063/5.0205643>
- [2] Javor, J., Stange, A., Pollock, C., Fuhr, N., & Bishop, D. J. (2019). 100pTcm Sensitive MEMS Resonant Magnetometer from a Commercial Accelerometer. *arXiv preprint arXiv:1911.10250*.
- [3] Liu, C. (2006). Foundations of MEMS.